Homework

October 22, 2019

1 Lecture 1

1. Let (u_i, v_i) be the data in a Machine Learning problem. Let the loss function be given by

$$L(\theta) = \sum_{i=1}^{m} \ell(u_i^T \theta, v_i),$$

where $\ell(\cdot, v)$ is a given function $\mathbb{R} \to \mathbb{R}$. Find the gradient and Hessian of this matrix. Provide a sufficient condition for $L(\theta)$ to be convex.

2. Consider a data model

$$v = u^T \theta + \xi,$$

where ξ has Laplace distribution `https://en.wikipedia.org/wiki/Laplace_distribution` with parameters $\mu = 0$, $b = 1$. Assume that we have access to an i.i.d. sample of size m. Show that this distribution belongs to the exponential family. Write the likelihood for this model. Show that it belongs to a class of generalized linear models. Write a log-likelihood maximization problem.

3. Is the set

$$\{x \in \mathbb{R}^n : x^T Q x \leq 1, Ax \leq b\}$$

convex? Under what conditions on the parameters Q, A, b is it convex?

4. For which values of parameter $p \in (0, +\infty]$ the set

$$\left\{ x \in \mathbb{R}^n : \left(\sum_{i=1}^{n} |x_i|^p \right)^{1/p} \leq 1 \right\}$$

is convex? Why? Note: By convention, for $p = +\infty$, $(\sum_{i=1}^{n} |x_i|^p)^{1/p} = \max_{i=1,\dots,n} |x_i|$.

5. Is the function

$$\max\{|x + y - 1|, |2x - y + 4|\}$$

convex? Why? Find its subdifferential at the point $x = -1$, $y = 2$.

6. Find the support function for the set

$$\{x \in \mathbb{R}^n : \|x - x_0\|_2 \leq R\}$$

where $R > 0$, $x_0 \in \mathbb{R}^n$ are parameters. Note: support function of a set Q is defined as

$$s(p, Q) = \max_{x \in Q} p^T x.$$